MATRIX

JEE MAIN 2026

Memory Based-Questions and Analysis of 2nd April (Shift-2)

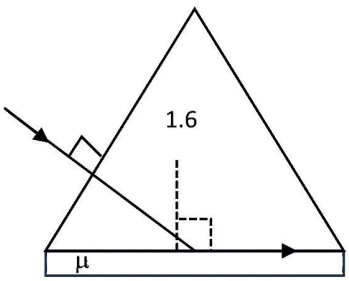
PHYSICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. In equilateral prism the path of a ray is shown. Determine is



(1) 1.71 (2) 1.52

(3) 1.39 (4) 1.84

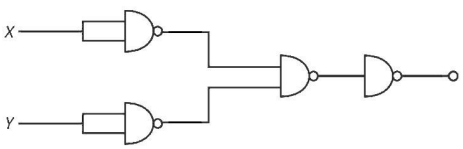
Answer (3)

Sol. $\sin 60° \, 1.6 = \mu \sin 90°$

$\Rightarrow \sqrt{3} \times 0.8 = \mu$

$\Rightarrow \mu = 1.39$

2. What is the equivalent gate for the circuit.



(1) AND gate (2) OR gate

(3) NAND gate (4) NOR gate

Answer (4)

Sol. $\overline{\overline{X} \cdot \overline{Y}} = X + Y$

Final output $\overline{X+Y}$

It is NOR gate

3. A soap bubble of radius r = 1mm, completely submerged in liquid of density $\rho_1 = 2000$ kg/m^3. At the instant bubble is rising upward with constant velocity $v = \frac{1}{2}$ cm/s. Find coefficient of viscosity (η).

(1) $\frac{2}{9}$ N-S/m^2 (2) $\frac{4}{9}$ N-S/m^2

(3) $\frac{2}{3}$ N-S/m^2 (4) $\frac{8}{9}$ N-S/m^2

Answer (4)

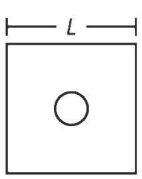
Sol. $6\pi\eta r v = \frac{4}{3}\pi r^3 \rho_L g$

$\Rightarrow \eta = \frac{4\pi r^3 \rho_L g}{3 \times 6\pi r v} = \frac{2}{9}\frac{r^2 \rho_L g}{v}$

$\Rightarrow \eta = \frac{2}{9} \times \frac{1 \times 10^{-6} \times 2000 \times 10 \times 2}{1 \times 10^{-2}}$

$\Rightarrow \eta = \frac{8}{9}$ N-s/m^2

4. A small circular loop of radius r is completely in closed within a large square loop of length L. Both of the loops are concentric and co-planer. (Also $L >> r$). Find co-efficient of mutual induction.



(1) $M = \frac{\mu_0 r^2 2\sqrt{2}}{L}$ (2) $M = \frac{\mu_0 L^2}{\sqrt{2} r}$

(3) $M = \frac{2\mu_0 r^2}{L}$ (4) $M = \frac{\mu_0 r^2}{2L}$

Answer (1)

Sol. $B=\frac{\mu_0 I\times 2}{4\pi L}\times 2\cdot\frac{1}{\sqrt{2}}\times 4=\frac{\mu_0 I\times 4}{\pi L\sqrt{2}}$

$\phi=\frac{(\pi r^2)\mu_0 I\times 4}{\pi L\sqrt{2}}$

So $M=\frac{\mu_0 r^2 2\sqrt{2}}{L}$

5. Dimensions of G (Universal gravitational constant) in terms of h (Plank's constant), m (mass) and t (time) and L (length) will be

(1) $h^{-1}L\,m^{-2}t$ (2) $h\,L^{-1}\,m^2\,t$

(3) $h\,L\,m^{-2}\,t^{-1}$ (4) $h^{-1}L^{-1}\,m^2\,t^{-1}$

Answer (3)

Sol. Planck mass $m^2=\frac{hc}{G}$

$G=\frac{hc}{m^2}=\frac{hL}{m^2t}$

6. Position of a particle is given by $x = a\sin(50t + \pi/3)$. If speed and acceleration becomes zero for the first time at time t_1 & t_2 respectively. Then t_1 & t_2 are

(1) $\frac{\pi}{150},\frac{\pi}{300}$ (2) $\frac{\pi}{300},\frac{\pi}{75}$

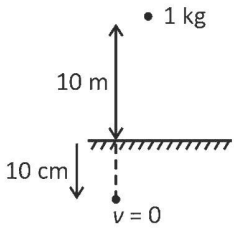
(3) $\frac{\pi}{50},\frac{\pi}{150}$ (4) $\frac{\pi}{300},\frac{\pi}{150}$

Answer (2)

Sol. $v=50a\cos\left(50t+\frac{\pi}{3}\right)$ $\quad v=0\Rightarrow t=\frac{\pi}{6\times 10}=\frac{\pi}{300}$

$a=50^2 a\sin\left(50t+\frac{\pi}{3}\right)$ $\quad a=0\Rightarrow t=\frac{2\pi}{3\times 50}=\frac{\pi}{75}$

7. A ball is released from rest as shown in figure. Ball comes to rest after moving 10 cm below the surface. Find average force applied by the floor material.



(1) 800 N (2) 1000 N

(3) 500 N (4) 1200 N

Answer (2)

Sol. $u=\sqrt{2\times 10\times 10}=10\sqrt{2}$ m/s

$v^2 = u^2 + 2as$

$0 = 200 + 2a \times 0.1$

$a = -1000$ m/s^2

$\bar{F}=m\bar{a}$

$= 1 \times 1000$

$= 1000$

8. In Bohr's atomic model. Find ratio of magnetic field produced at center by electron in 2nd orbit and 4th orbit.

(1) 8 (2) 4

(3) 32 (4) 16

Answer (3)

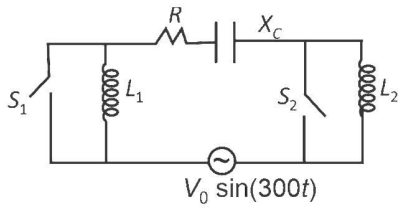
Sol. We know $r_{(n)}=\frac{r_0 n^2}{Z}$ & $v=v_0\frac{Z}{n}$

So $I=\frac{ev}{2\pi r}$

$\beta=\frac{\mu_0 I}{2r}=\frac{\mu_0}{2r}\frac{ev}{2\pi r}=\frac{\mu_0 ev}{4\pi r^2}$

Clearly $\beta\propto\left(\frac{Z}{\eta}\right).\frac{Z^2}{\eta^4}\Rightarrow\beta\propto\frac{Z^3}{\eta^5}$

So $\frac{\beta_{(2)}}{\beta_{(4)}}=\left(\frac{4}{2}\right)^5=32$

9. 

In a given AC circuit if only switch S_1 is closed then phase difference is 30° while only if only switch S_2 is closed than phase difference is 60°. Current lags in both cases. If X_C = 30 W then $3L_1 - L_2$ is

(1) 0.3 H

(2) 0.5 H

(3) 0.2 H

(4) 10 H

Answer (3)

Sol. $\frac{1}{\sqrt{3}} = \frac{L_1\omega - X_C}{R}$

$\sqrt{3} = \frac{L_2\omega - X_C}{R}$

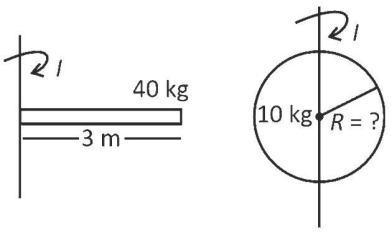
$\Rightarrow \frac{1}{3} = \frac{L_1\omega - X_C}{L_2\omega - X_C}$

$3L_1\omega - 3X_C = L_2\omega - X_C$

$3L_1\omega - L_2\omega = 2X_C$

$3L_1 - L_2 = \frac{2X_C}{300} = \frac{X_C}{150} = \frac{30}{150} = 0.2$

10. A rod of length 3 m and ass 40 kg has same moment of inertia as that of solid sphere. Find radius of solid sphere.



(1) $\frac{5}{2}$

(2) $\sqrt{30}$

(3) $\frac{3}{2}$

(4) $\sqrt{5}$

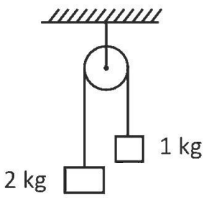
Answer (2)

Sol. $\frac{ml^2}{3} = \frac{2}{5}MR^2$

$\Rightarrow \frac{40\times 9}{3} = \frac{2}{5}\times 10R^2$

$\Rightarrow R = \sqrt{30}$

11. Find displacement of center of mass after t = 2 sec.



(1) $\frac{20}{9}$

(2) $\frac{10}{9}$

(3) $\frac{25}{9}$

(4) $\frac{5}{9}$

Answer (1)

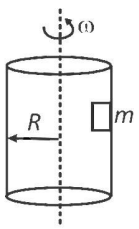
Sol. $a = \frac{2g - g}{3} = \frac{g}{3}$

So $x = \frac{1}{2}\times\frac{g}{3}\times 4 = \frac{2g}{3}$

So shifting of COM

$\Delta y = \frac{2\left(\frac{2g}{3}\right) - 1\left(\frac{2g}{3}\right)}{3} = \frac{2g}{9} = \frac{20}{9}$

12. In the situation shown a small mass 'm' is rotating along with the cylinder with angular speed $\omega = 5$ rad/s. Radius of the cylinder is $R = \frac{1}{2}$ m. Find the minimum value of friction coefficient between inner surface of cylinder and the mass 'm' so that it does hot slip.



(1) $\frac{5}{4}$ (2) $\frac{5}{2}$

(3) $\frac{2}{5}$ (4) $\frac{4}{5}$

Answer (4)

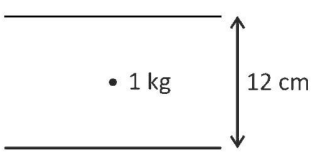
Sol. $N = m\omega^2 R$

So $\mu m\omega^2 R \geq mg$

$\Rightarrow \mu m\omega^2 R \geq mg$

$\Rightarrow \mu \geq \frac{g}{\omega^2 R} = \frac{10\times 2}{5\times 5\times 1} = \frac{4}{5}$

13. A ball of mass 1 kg and charge 9 nC is in equilibrium between two parallel plates. Find potential difference between the plates.



(1) 24×10^3 V (2) $\frac{4}{3}\times 10^8$ V

(3) 6×10^6 V (4) 12×10^5 V

Answer (2)

Sol. $qE = mg$

$$9\times 10^{-9} \times \frac{V}{12\times 10^{-2}} = 10$$

$$V = \frac{4}{3}\times 10^8$$

14. A point mars just starts appropriating earth from a large distance. Speed of impact of point of mars to planet's surface is _____. [Planet is a times heavier and with radius 4 times compared to earth]

(1) 25 km/s (2) 10 km/s

(3) 17 km/s (4) 5 km/s

Answer (3)

Sol. $\frac{GMm}{R} = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{2GM}{R}}$$

$$v = 11.2\times \sqrt{\frac{9}{4}}$$

15. Find the work done when radius of a soap bubble is increased from 2 cm to 6 cm. [Surface tension of soap 0.03 N/m.]

(1) 6 mJ

(2) 2.4 mJ

(3) 3.6 mJ

(4) 2.1 mJ

Answer (2)

Sol. $W = \Delta V = S[4\pi\left(R_2^2 - R_1^2\right)\times 2$

$= 0.03\times 4\pi\times(36-4)\times 10^{-4}\times 2$

$= 3.84\pi\times 10^{-4}\times 2$

$= 2.4$ mJ

16. Temperature of a gas is increased from 10 k to 20 k isochorically. Find heat absorbed by the 5 moles of the gas. (C_p = 8 cal/mol k)

(1) 340 J (2) 120 J

(3) 520 J (4) 1260 J

Answer (4)

Sol. $\phi = 5C_v\Delta T$

$= 50 \times 6 \times 4.2$

$= 1260$ J

17. A mixture of 13.2 gram of CO_2 & O_2 gas is maintained at pressure = 100 kpa, volume $v = 8310\ cm^3$ and temperature $T = 300$ K. Find number of moles of CO_2 & O_2 respectively

(1) $\frac{2}{9}, \frac{1}{9}$ (2) $\frac{1.9}{9}, \frac{1.1}{9}$

(3) $\frac{1.6}{9}, \frac{1.4}{9}$ (4) $\frac{1.2}{9}, \frac{1.8}{9}$

Answer (2)

Sol. $NRT = 100 \times 10^3 \times 8.31 \times 10^3 \times 10^{-6}$

$\Rightarrow N = \frac{8.31 \times 100}{8.31 \times 300} = \frac{1}{3}$

Also, $N_1(44) + N_2(32) = 13.2$

$N_2 = \left(\frac{1}{3} - N_1\right)$

$44N_1 + \frac{32}{3} - 32N_1 = 13.2$

$\Rightarrow 12N_1 = \frac{7.6}{3} \Rightarrow N_1(CO_2) = \frac{1.9}{9}$ moles

And $N_2 = \frac{1}{3} - \frac{1.9}{9} = \frac{1.1}{9}$ mole

18.

19.

20.

SECTION - B

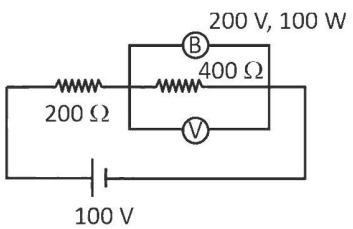
Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. Two point charges $q_1 = +8\mu C$ and $q_2 = -2\mu C$ are placed at $x = 2$ and $x = 4$ respectively. Consider two spherical Gaussian surfaces S_1 and S_2 centered at origin of radius 3 m and 5 m respectively. Electric flux through S_1 and S_2 is ϕ_1 and ϕ_2 then, $\frac{3\phi_1}{\phi_2}$ _____

Answer (4)

Sol. $\frac{\phi_1}{\phi_2} = \frac{8\varepsilon_0}{\varepsilon_0(8-2)} = \frac{8}{6} = \frac{4}{3}$

22. In a DC circuit a bulb (200 V, 100 W) is connected with 2 resistances and an ideal voltmeter as shown. Reading of voltmeter is V volt then V is



Answer (50)

Sol. $R = \frac{V^2}{P} = \frac{200 \times 200}{100} = 400$

$R_{eq} = 200 + 400 \,||\, 400$

$\Rightarrow V_B = V_{200} \Rightarrow V_B = V_v = 50$ V

23. The intensity of sunlight, collected from equiconvex lens of $R = 60$ cm, is observed to be maximum at distance of 30 cm from lens, then refractive index of material of lens is

Answer (2)

Sol. $\frac{1}{f} = (\mu - 1)\frac{2}{R} \Rightarrow \frac{1}{30} = (\mu - 1)\frac{2}{60}$

$\Rightarrow \mu = 2$

24.

25.

CHEMISTRY

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer:

1. Arrange following complexes in increasing order of CFSE (Δ_o)

   (a) $[Co(H_2O)_6]^{2+}$

   (b) $[Co(H_2O)_6]^{3+}$

   (c) $[Co(en)_3]^{3+}$

   (1) c > a > b

   (2) c > b > a

   (3) a > b > c

   (4) b > a > c

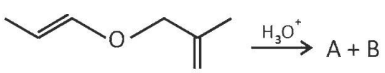
Answer (2)

Sol. H_2O is SFL with Co^{3+}, en is SFL with Co^{3+}.

Order of ligand strength en > H_2O

CFSE (Δ_o) order $\Rightarrow$ c > b > a

2. Consider the following reaction



A gives positive fehling's test. Choose the correct relation.

(1) Molar weight of A and molar weight of B is same

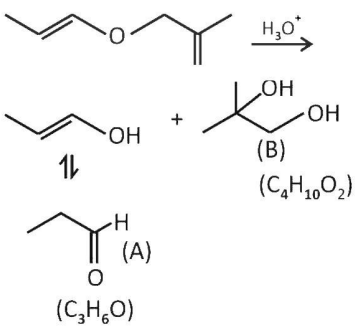
(2) Molar weight of A is greater than molar weight of B

(3) Molar weight of B is greater than molar weight of A

(4) None

Answer (3)

Sol.



3. 20 mL of 0.2 M HA (weak monoprotic acid) is titrated with 10 mL of 0.2 M NaOH solution. pH of solution at 25°C is, (pK_a of weak acid is 4.76)

   (1) 9.24 (2) 5.24

   (3) 4.76 (4) 9.76

Answer (3)

Sol. $HA + NaOH \rightarrow NaA + H_2O$

20×0.2	10×0.2	
4	2	
2	0	2

$$pH = pk_a + \log\frac{2/V}{2/V}$$

$pH = pk_a = 4.76$

4. Molarity of H_2SO_4 solution is 4.9 M. If density of solution is 1.40 g/ml, then molality and mole fraction of solute in solution is

   (1) m = 5.34, $\chi_{solute} = 0.088$

   (2) m = 5.34, $\chi_{solute} = 0.072$

   (3) m = 5.21, $\chi_{solute} = 0.088$

   (4) m = 5.21, $\chi_{solute} = 0.072$

Answer (1)

Sol. Let volume of solution = 1000 ml, $W_{solution}$ = 1000 × 1.4 = 1400 g

$W_{solvent} = 1400 - (4.9 \times 98) = 919.8$ g, $n_{solute} = 4.9$

$$m = \frac{4.9}{919.8 \times 10^{-3}} = 5.34 \text{ mol/kg}$$

$$\chi_{solute} = \frac{4.9}{(4.9) + \left(\frac{919.8}{18}\right)} = 0.088$$

5. SF_4 is isostructural with

(I) XeO_2F_2

(II) CH_4

(III) IF_4^+

(IV) BrF_4^-

(1) (I), (III), (IV) only (2) (I), (III) only

(3) (II), (IV) only (4) (I), (II), (III), (IV)

Answer (2)

Sol. $SF_4 \Rightarrow$ see-saw shape (sp^3d)

$XeO_2F_2 \Rightarrow$ see-saw shape (sp^3d)

$CH_4 \Rightarrow$ Tetrahedral (sp^3)

$IF_4^+ \Rightarrow$ see-saw shape (sp^3d)

$BrF_4^- \Rightarrow$ square planar (sp^3d^2)

6. For the reaction $2A \rightarrow 4B + C$. At 30 minutes the total pressure is 300 mm of Hg and after infinite time the total pressure is 600 mm of Hg. Then the pressure of C at t = 30 minutes will be

(1) 20 mm Hg (2) 40 mm Hg

(3) 60 mm Hg (4) 10 mm Hg

Answer (1)

Sol.

	2A	$\rightarrow$	4B	+	C
t = 0	P_0		0		0
t = t	$P_0 - 2p$		4p		p
$t = \infty$	0		$2P_0$		$\frac{P_0}{2}$

At $t = \infty$

total pressure = 600 mm of Hg

$$2P_0 + \frac{P_0}{2} = 600$$

$$\frac{5P_0}{2} = 600$$

$P_0 = 240$ mm of Hg

At t = 30 minutes

total pressure = $P_0 - 2p + 4p + p = 300$

$P_0 + 3p = 300$

$3p = 300 - P_0$

$3p = 300 - 240$

$3p = 60$

$p = 20$ mm of Hg

Pressure of C at t = 30 minutes = 20 mm of Hg

7. Two solutions of protein (M.Wt = 50,000 g/mol) are prepared separately

Solution A : 1 g protein in 0.5 L solution

Solution B : 2 g protein in 1.0 L solution

When these two solutions are mixed at 300 K. Find total osmotic pressure :

(R = 0.08 L-atm K^{-1} mol^{-1})

(1) 9.8×10^{-3} torr

(2) 6.5×10^{-2} torr

(3) 7.3×10^{-1} torr

(4) 5.4×10^{-4} torr

Answer (3)

Sol. $$\pi_{total} = \frac{1}{5 \times 10^4}\left[\frac{1}{1.5} + \frac{2}{1.5}\right] \times 0.08 \times 300$$

$$= \frac{1}{5 \times 10^4}\left[\frac{3}{1.5}\right] \times 0.08 \times 300$$

$= 9.6 \times 10^{-4}$ atm

$= 7.296 \times 10^{-1}$ torr

8. Given below are two statements

Statement I : Order of second ionisation energy is B > Al > Ga

Statement II : Order of first ionisation energy is B > Ga > Tl > Al > In

(1) Both statement I and statement II are correct

(2) Statement I is correct but statement II is incorrect

(3) Statement I is incorrect but statement II is correct

(4) Both statement I and statement II are incorrect

Answer (4)

Sol.

(kJ/mol)	B	Al	Ga	In	Tl
$I.E_1$:	801	577	579	558	589
$I.E_2$:	2427	1816	1979	1820	1971

9. Correct order of enthalpy of atomisation is,

(1) $N_2 > O_2 > F_2 > Cl_2 > Br_2 > I_2$

(2) $N_2 > O_2 > Cl_2 > F_2 > Br_2 > I_2$

(3) $N_2 > O_2 > Cl_2 > Br_2 > F_2 > I_2$

(4) $N_2 > O_2 > F_2 > Br_2 > Cl_2 > I_2$

Answer (3)

Sol.

	Bond energy (KJ/mol)
Cl_2	242.6
Br_2	192.8
F_2	158.8
I_2	151.1

10. Which of the ion having highest ionisation energy will give borax bead test.

(1) Fe^{2+}

(2) Fe^{3+}

(3) Cr^{3+}

(4) Zn^{2+}

Answer (2)

Sol. $Fe^{3+} \Rightarrow 3d^5$

$Cr^{3+} \Rightarrow 3d^3$

$\therefore$ Fe^{3+} has higher ionisation energy than Cr^{3+}

$\therefore$ Fe^{3+} and Cr^{3+} both give borax bead test.

11. Consider the reaction, $aX \rightarrow bY$ –(1)

Rate constant $k = 3 \times 10^{-4}\ M^{-1}\ sec^{-1}$

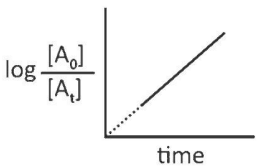
The correct statement is

(1) $t_{1/2}$ is independent of initial concentration. of 'X'

(2) Dissociation of N_2O_5 is an example of reaction

(3) When conc. of 'X' becomes 4 times then rate of reaction increases by 16 times

(4) The graph between $\log\frac{[A_0]}{[A_t]}$ v/s time is



Answer (3)

Sol. $aX \rightarrow bY$

$k = 3 \times 10^{-4}\ M^{-1}s^{-1}$

(second order reaction)

$r = k[X]^2$

$$\frac{r_1}{r_2} = \left[\frac{X_1}{X_2}\right]^2 = \left(\frac{1}{4}\right)^2$$

$16r_1 = r_2$

12. Consider the following reaction

$$X \xrightarrow{\text{aromatisation}} \text{Toluene}$$

$$X \xrightarrow{AlCl_3(anhy)/HCl} 3-\text{methylhexane}$$

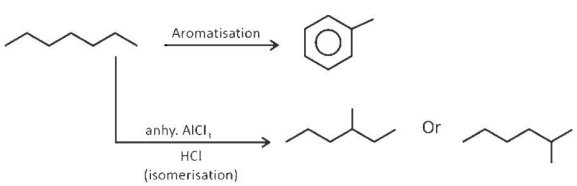
X may be

(1) 2-methylhexane (2) Heptane

(3) 3-methylhexane (4) Octane

Answer (2)

Sol.



13. Correct order of boiling point of Si, Ge, Sn and Pb is

(1) Si > Ge > Sn > Pb

(2) Pb > Sn > Ge > Si

(3) Si > Ge > Pb > Sn

(4) Pb > Ge > Sn > Si

Answer (1)

Sol. Boiling point of Si = 3550 K

Ge = 3123 K

Sn = 2896 K

Pb = 2024 K

14. Which option have acidic, basic, amphoteric and neutral oxide respectively?

(1) Na_2O, Al_2O_3, MgO, As_2O_3

(2) CO_2, Na_2O, Al_2O_3, CO

(3) As_2O_3, Al_2O_3, Na_2O, CO

(4) None of these

Answer (2)

Sol. $CO_2 \rightarrow$ acidic

$Na_2O \rightarrow$ basic

$Al_2O_3 \rightarrow$ amphoteric

$CO \rightarrow$ neutral

15. The correct order of reactivity of SN_1 is

(1) $C_6H_5C(CH_3)C_6H_5Br > C_6H_5CH_2Br > C_6H_5CHC_6H_5Br$

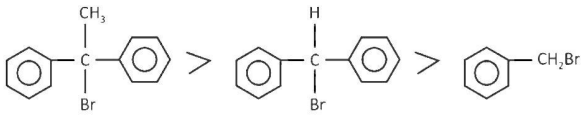
(2) $C_6H_5C(CH_3)(C_6H_5)Br > C_6H_5CHC_6H_5Br > C_6H_5CH_2Br$

(3) $C_6H_5CHC_6H_5Br > C_6H_5CH_2Br > C_6H_5C(CH_3)C_6H_5Br$

(4) $C_6H_5CH_2Br > C_6H_5CHC_6H_5Br > C_6H_5C(CH_3)C_6H_5Br$

Answer (2)

Sol. The rate of SN_1 reaction will depend on stability of carbocation



16. The thyroxine hormone is formed by the iodination of an alpha amino acids. The single letter code of that amino acid is

(1) Y

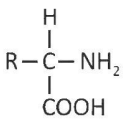
(2) T

(3) W

(4) H

Answer (1)

Sol. Thyroxine is formed by the iodination of tyrosine (Y)



R = (p) OH – C_6H_4 – CH_2 –

17.

18.

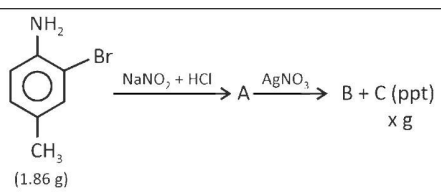
19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

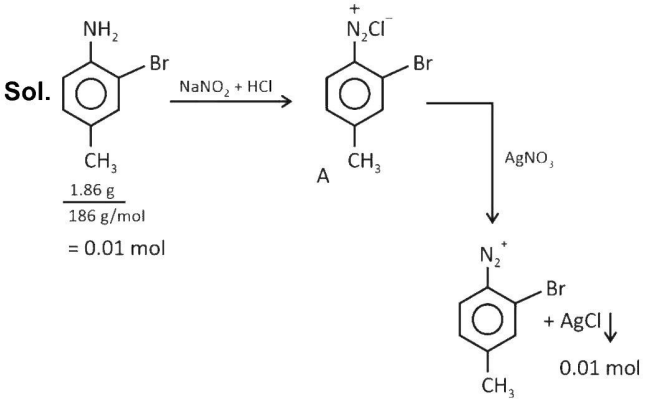
21.



'C' is a white curdy ppt. Find 10(x) (Nearest integer)

Answer (14)

Sol.



Molecular weight of the reactant

= 75 + 16 + 80 + 15 = 186 g

Mass of 'C' (x) = 0.01 × 143.5 = 1.435 g

22. The work function of Na metal is 2.3 eV. If maximum kinetic energy of emitted photoelectron is 2.8×10^{-19} J, then calculate wavelength of incident photon in nm. ($h = 6.626 \times 10^{-34}$ J-sec)

Answer (308)

Sol. $\frac{hc}{\lambda} = \phi + KE_{max}$

$\frac{hc}{\lambda} = 2.3 \times 1.6 \times 10^{-19} + 2.8 \times 10^{-19}$

$\frac{hc}{\lambda} = (3.68 + 2.8) \times 10^{-19}$

$\frac{hc}{\lambda} = 6.48 \times 10^{-19}$

$\lambda = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{6.48 \times 10^{-19}}$

$\lambda = 3.068 \times 10^{-7}$

$\lambda = 306.8 \times 10^{-9}$ m

$\lambda = 306.8$ nm ≈ 307 nm

23. How many of following ions will show paramagnetism?

$Mn^{++}, Cu^{++}, Zn^{++}, Sc^{3+}, Ti^{++}, Yb^{++}, Gd^{++}, Lu^{3+}, Ce^{3+}, La^{3+}$

Answer (5)

Sol. $Mn^{2+} \rightarrow 3d^5$ (Para)

$Cu^{2+} \rightarrow 3d^9$ (Para)

$Zn^{2+} \rightarrow 3d^{10}$ (Dia)

$Sc^{3+} \rightarrow 3d^0$ (Dia)

$Ti^{2+} \rightarrow 3d^2$ (Para)

$Yb^{2+} \rightarrow 4f^{14}$ (Dia)

$Gd^{4+} \rightarrow 4f^6$ (Para)

$Lu^{3+} \rightarrow 4f^{14}$ (Dia)

$Ce^{3+} \rightarrow 4f^1$ (Para)

$La^{3+} \rightarrow 4f^0$ (Dia)

Number of paramagnetic ions is 5

24. The $[HA^-]$ concentration in a mixture containing 0.1 M H_2A and 0.1 M HCl is $x \times 10^{-9}$. Value of x is

Given : $Ka_1 = 10^{-8}$ and $Ka_2 = 10^{-13}$ for H_2A

Answer (10)

Sol. $HCl \rightarrow \underset{0.1+0.1\alpha}{H^+} + \underset{0.1}{Cl^-}$

For H_2A $Ka_1 >> Ka_2$, so second dissociation can be neglected

$\underset{0.1(1-\alpha)}{H_2A} \rightleftharpoons \underset{0.1+0.1\alpha}{H^+} + \underset{0.1\alpha}{HA^-}$

$Ka_1 = \frac{[H^+][HA^-]}{[H_2A]}$

$10^{-8} = \frac{(0.1+0.1\alpha)[HA^-]}{0.1(1-\alpha)}$

$\alpha << 1$

$10^{-8} = [HA^-]$

25.

MATHEMATICS

SECTION - A

Multiple Choice Questions: This section contains 20 multiple choice questions. Each question has 4 choices (1), (2), (3) and (4), out of which **ONLY ONE** is correct.

Choose the correct answer :

1. The value of $\int_0^{20\pi} (\sin^4 x + \cos^4 x)dx$ is equal to

(1) 15π (2) 10π

(3) 20π (4) 40π

Answer (1)

Sol. $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x$

$$= 1 - \frac{\sin^2(2x)}{2} = 1 - \frac{1-\cos(4x)}{4} = \left(\frac{3}{4} + \frac{\cos(4x)}{4}\right)$$

$$I = \int_0^{2\pi} (\sin^4 x + \cos^4 x)dx$$

$$= 40\int_0^{\frac{\pi}{2}} \left(1 - \frac{1}{2}\sin^2 x\right)dx$$

$$= 40\int_0^{\frac{\pi}{2}} \left(1 - \frac{1-\cos(4x)}{4}\right)dx$$

$$= 40\int_0^{\frac{\pi}{2}} \left(\frac{3}{4} + \frac{\cos(4x)}{4}\right)dx$$

$$= 40\left[\frac{3x}{4} + \frac{\sin(4x)}{16}\right]_0^{\frac{\pi}{2}}$$

$$= 40\left(\frac{3\pi}{8} + 0 - 0 - 0\right) = 15\pi$$

2. If $A = \begin{pmatrix} 2 & -2 \\ 4 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 3 \\ 1 & 9 \end{pmatrix}$ and $PA = B$ and $AQ = B$, then $2(P + Q)$ is

(1) $\begin{pmatrix} 11 & 12 \\ 24 & 13 \end{pmatrix}$ (2) $\begin{pmatrix} -11 & 12 \\ 24 & 13 \end{pmatrix}$

(3) $\begin{pmatrix} -11 & 12 \\ -24 & 13 \end{pmatrix}$ (4) $\begin{pmatrix} 11 & 12 \\ -24 & 13 \end{pmatrix}$

Answer (3)

Sol. $\because PA = B$

$P = BA^{-1}$

$$\begin{bmatrix} 3 & 3 \\ 1 & 9 \end{bmatrix}\begin{bmatrix} -\frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} \end{bmatrix}$$

$$P = \begin{pmatrix} -\frac{9}{2} & 3 \\ -\frac{19}{2} & 5 \end{pmatrix}$$

$\because AQ = B$

$Q = A^{-1}B$

$$= \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ -1 & \frac{1}{2} \end{pmatrix}\begin{pmatrix} 3 & 3 \\ 1 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} -1 & 3 \\ -\frac{5}{2} & \frac{13}{2} \end{pmatrix}$$

Now, $2(P+Q) = 2\begin{pmatrix} -\frac{11}{2} & 6 \\ -12 & \frac{13}{2} \end{pmatrix}$

$$= \begin{pmatrix} -11 & 12 \\ -24 & 13 \end{pmatrix}$$

3. If $f(x)=\int\frac{16x+24}{x^2+2x-15}dx$, given that $f(4) = 14\ln 3$, then the value of f(7) is

(1) $32\ln 2 + 7\ln 3$ (2) $7\ln 2 + 32\ln 3$

(3) $32\ln 2 - 7\ln 3$ (4) $32\ln 3 - 7\ln 2$

Answer (1)

Sol. $f(x)=\int\frac{16x+24}{x^2+2x-15}dx$

$\because f(x)=\frac{16x+24}{x^2+2x-15}=\frac{16x+24}{(x+5)(x-3)}$

$=\frac{7}{x+5}+\frac{9}{x+3}$

$\Rightarrow f(x)=\int\left(\frac{7}{x+5}+\frac{9}{x-3}\right)dx$

$\Rightarrow f(x)=7\ln|x+5|+9\ln|x-3|+c$

$\because f(4)=14\ln 3$

$\Rightarrow 14\ln 3=14\ln 3+c\Rightarrow c=0$

$\Rightarrow f(x)=7\ln|x+5|+9\ln|x-3|$

$\Rightarrow f(7)=7\ln 12+9\ln 4=32\ln 2+7\ln 3$

4. Let $A = \{2, 3, 4,5, 6\}$, consider R be relation of $A \times A$ such that $(x, y)\ R\ (a, b)$ implies that x divides a and $y \le b$ then total number of elements in R is

(1) 24 (2) 120

(3) 720 (4) 144

Answer (2)

Sol. $A=\{2,3,7,5,6\}$ $(x,y)\in A\times A$

$(a,b)\in A\times A$ $(x, y)\ R\ (a, b)$

$\Rightarrow x \mid a$ and $y \le b$

$(A\times A)=\{(2,2)\,(2,3)\,(2,4)\,(2,5)\,(2,6)$

$(3,2)\,(3,3)\,(3,4)\,(3,5)\,(3,6)$

$(4,2),(4,3)\,(4,4)\,(4,5)\,(4,6)$

$(5,2)\,(5,3)\,(5,4)\,(5,5)\,(5,6)$

$(6,2)\,(6,3)\,(6,4)\,(6,5)\,(6,6)\}$

Consider $x=2\Rightarrow a$ can be $\{2,4,6\}$

$\Rightarrow$ (3 choices) $\times$ (15) $= 45$

$x = 3, a$ can be $\{3,6\}$

$\Rightarrow 2\times 15 = 30$

$x = 4, a$ can be $\{4\}\Rightarrow 1\times 15$

$x = 5, a$ can be $\{5\} = 1\times 15$

$x = 6, a$ can be $\{6\} = 1\times 15$

= 120

5. Let a_1, a_2, a_3..... be an arithmetic progression and g_1, g_2, g_3.... be an increasing geometric progression such that $g_1 = a_1 = a_2 + g_2 = 1$ and $g_3 + a_3 = 4$ then $a_{10} + g_5$ is equal to

(1) 34

(2) 35

(3) 55

(4) 54

Answer (3)

Sol. $a_1 = 1$, $a_2 + g_2 = a + d + br = 1$

$d = -br$

$a_3 + g_3 = (a + 2d) + br^2 = 4$

$\Rightarrow$ $1 + 2d + br^2 = 4$

$\Rightarrow$ $br^2 - 2br = 3$

$r^2 - 2r - 3 = 0$, $(b = 1)$

$(r - 3)\ (r + 1) = 0$

$r = 3$

$d = -3$

$a_{10} + g_5 = (a + 9d) + (br^4)$

$= 1 + 9(-3) + (1)(3^4)$

$= 1 - 27 + 81$

$= 82 - 27 = 55$

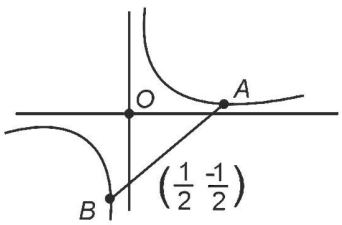
6. If AB is a chord of hyperbola $xy = 12$, whose mid-point is $\left(\frac{1}{2}, \frac{-1}{2}\right)$, then the area of triangle ABO (where O is the origin) is

(1) $\frac{7}{2}$ (2) $\frac{9}{2}$

(3) 7 (4) $\frac{3}{2}$

Answer (1)

Sol.



$xy = 12$, for chord with mid point

$T = S_1$

$$\Rightarrow \frac{xy_1 + yx_1}{2} - 12 = x_1y_1 - 12$$

$$\Rightarrow \frac{x\left(-\frac{1}{2}\right) + \left(\frac{1}{2}\right)}{2} = \frac{-1}{4}$$

$\Rightarrow y - x + 1 = 0$

$$xy = 12 \Rightarrow y = x - 1 = \frac{12}{x}$$

$\Rightarrow x^2 - x + 12 = 0$

$(x - 7)(x + 3) = 0$

$x = -3, 4$

$\Rightarrow$ Point A and B are $(-3, -4)$ and $(4, 3)$

$$\text{Area} = \left|\frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -3 & -4 & 1 \\ 4 & 3 & 1 \end{vmatrix}\right|$$

$$= \frac{1}{2}(-9 + 16) = \frac{7}{2} = 3.5$$

7. Let $\vec{a} = -\hat{i} + 2\hat{j} + \hat{k}$

$\vec{b} = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{c} = \lambda\vec{a} + \mu\vec{b}$ and

$\vec{c} \cdot (3\hat{i} - 6\hat{j} + 3\hat{k}) = 10$

$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$

Then $|\vec{c}| =$

(1) $\frac{\sqrt{1920}}{18}$ (2) $\frac{\sqrt{1914}}{18}$

(3) $\frac{\sqrt{920}}{18}$ (4) $\frac{\sqrt{914}}{18}$

Answer (2)

Sol. $a = -\hat{i} + 2\hat{j} + \hat{k}$

$b = \hat{i} + \hat{j} - 3\hat{k}$

$\vec{c} = \lambda\vec{a} + \mu\vec{b}$

$\Rightarrow \vec{c} = \lambda(-\hat{i} + 2\hat{j} + \hat{k}) + \mu(\hat{i} + \hat{j} - 3\hat{k})$

$= (-\lambda + \mu)\hat{i} + (2\lambda + \mu)\hat{j} + (\lambda - 3\mu)\hat{k}$

Now

$\vec{c} \cdot (3\hat{i} - 6\hat{j} + 3\hat{k}) = 10$

$\Rightarrow 3(-\lambda + \mu) - 6(2\lambda + \mu) + 3(\lambda - 3\mu) = 10$ ...(1)

Also

$\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = -2$

$\Rightarrow (-\lambda + \mu) + (2\lambda + \mu) + (\lambda - 3\mu) = -2$ ...(2)

Solving (1) and (2)

$$\lambda = \frac{-17}{18} \quad \mu = \frac{1}{9}$$

$$\vec{c} = \frac{19}{18}\hat{i} - \frac{16}{9}\hat{j} - \frac{23}{18}\hat{k}$$

$$|\vec{c}| = \sqrt{\frac{1}{18^2}(19^2 + 32^2 + 23^2)} = \frac{\sqrt{1914}}{18}$$

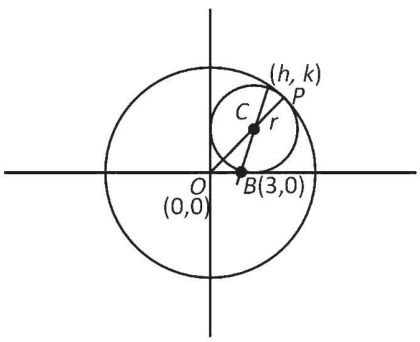
8. A variable circle with diameter AB where $A(3, 0)$ touching the circle $x^2 + y^2 = 36$ internally. The locus of centre of variable circle is a conic whose eccentricity is e, then $72e^2$ is

(1) 36 (2) 18

(3) 54 (4) 16

Answer (2)

Sol.



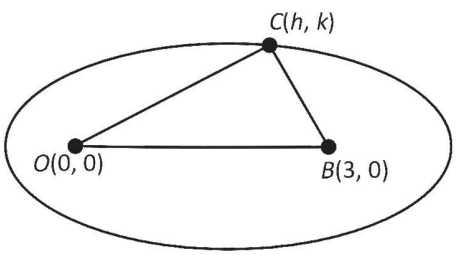
Let the centre be (h, k), then

$OP = 6, \quad OC = \sqrt{h^2 + k^2}, \quad CP = r = \sqrt{(h-3)^2 + y^2}$

$\Rightarrow OP = OC + CP$

$6 = \sqrt{h^2 + k^2} + \sqrt{(h-3)^2 + k^2}$

$\Rightarrow$ This is an ellipse



Such that: $OB = 2ae = 3$

$OC + CB = 6 = 2a$

$\Rightarrow a = 3, \quad e = \frac{1}{2}$

$\Rightarrow 72e^2 = 72 \times \frac{1}{4} = 18$

9. If $P = \{\theta \in [0, 4\pi] : \tan^2\theta \neq 1,$ $2(\cos^8\theta - \sin^8\theta)\sec 2\theta = a^2, a \in Z\}$ then the number of elements in set P is

(1) 4 (2) 3

(3) 2 (4) 0

Answer (4)

Sol. $a^2 = 2(\cos^8\theta - \sin^8\theta)\sec 2\theta$

$= 2(\cos^4\theta + \sin^4\theta)(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)\sec 2\theta$

$= 2(\cos^4\theta + \sin^4\theta)$

$= 2[(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta]$

$= 2 - \sin^2 2\theta$

$\because a \in Z \Rightarrow a^2 \in \{0, 1, 4, 9, ...\}$

$\Rightarrow (2 - \sin^2 2\theta) \in \{0, 1, 4, 9...\}$

$\Rightarrow \sin^2 2\theta = 1$

$\Rightarrow 2\theta = n\pi \pm \frac{\pi}{2}, n \in Z$

$\Rightarrow \theta = \frac{n\pi}{2} \pm \frac{\pi}{4}, n \in Z$

When $\theta = \frac{n\pi}{2} \pm \frac{\pi}{4}, n \in Z$, then $\tan^2\theta = 1$

$\because \tan^2\theta \neq 1$

$\Rightarrow P$ is null set

$\Rightarrow n(P) = 0$

10. If complex numbers z_1 and z_2 such that $|z_1| = 17$ and $|\bar{z}_2 - 3 - 4i| = 5$ then $\max(|z_1 + z_2|)$ is equal to

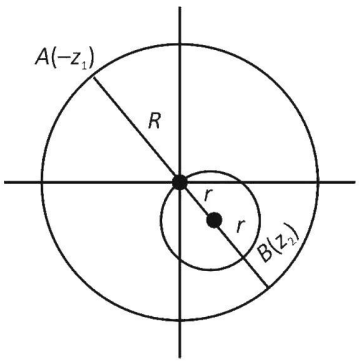
(1) 17 (2) 27

(3) 12 (4) 7

Answer (2)

Sol. $|\bar{z}_2 - (3 + 4i)| = 5$

$\Rightarrow |z_2 - (3 - 4i)| = 5$



$\Rightarrow$ maximum distance $AB = |z_2 - (-z_1)|$

$= R + 2r$

$= 17 + 2 \times 5 = 27$

11. A fair coin is tossed four times such that for heads 10 points are allotted and tail gets 5 points then probability that total points are 30 points is equal to

(1) $\frac{5}{8}$ (2) $\frac{1}{2}$

(3) $\frac{3}{5}$ (4) $\frac{3}{8}$

Answer (4)

Sol. Let a heads and $(4 - a)$ tails

$\Rightarrow$ $10a + 5(4 - a) = 30$

$\Rightarrow$ $20 + 5a = 30 \Rightarrow a = 2$

$\Rightarrow$ Probability = ${}^4C_2 \times \left(\frac{1}{2}\right)^4$

$= \frac{6}{16} = \frac{3}{8}$

12. If roots of the equation $x^2 - 3x + n = 0$ are α and β and equation $x^2 + 3x + 4 = 0$ are $\frac{\alpha}{3}, 3\beta$ then $(2\alpha + \beta + 3n)$ is equal to

(1) $\frac{47}{4}$ (2) $\frac{-51}{4}$

(3) $\frac{-37}{4}$ (4) $\frac{-25}{4}$

Answer (2)

Sol. $\alpha + \beta = 3$

$$\frac{\alpha}{3} + 3\beta = -3$$

$$\Rightarrow \alpha + \frac{\alpha}{3} + \beta + 3\beta = 0$$

$$-\frac{4\alpha}{3} = 4\beta \Rightarrow \alpha = -3\beta$$

$$\alpha + \beta = 3 \Rightarrow -2\beta = 3 \Rightarrow \beta = \frac{-3}{2}$$

$$\alpha = \frac{9}{2}$$

$$\alpha\beta = n = \frac{-27}{4}$$

$$\Rightarrow 2\alpha + \beta + 3n$$

$$= 2\left(\frac{9}{2}\right) + \left(\frac{-3}{2}\right) + \left(\frac{-81}{4}\right)$$

$$= \frac{9 - (81 + 6)}{4} = \frac{36 - 87}{4} = \frac{-51}{4}$$

13. If parabola $y = x^2 + px + q$ passes through $(1, -1)$ and whose vertex is at the minimum distance from x-axis then $p^2 + q^2$ is equal to

(1) 2 (2) 3

(3) 4 (4) 5

Answer (3)

Sol. $-1 = 1 + P + q \Rightarrow P + q = -2$

vertex is $\left(-\frac{P}{2}, -\left(\frac{P^2 - 4q}{4}\right)\right)$

$$= \left(-\frac{P}{2}, \frac{4q - P^2}{4}\right)$$

To minimise:

$$\frac{4q - P^2}{4} = \frac{4q - (-2 - q)^2}{4}$$

$$=\frac{4q-(q^2+4q+4)}{4}$$

$$=\frac{-q^2-4}{4}$$

$$\text{distance} = \left|\frac{-q^2-4}{4}\right| = \frac{q^2+4}{4} \Rightarrow q = 0$$

and $P = -2$

$$\Rightarrow P^2 + q^2 = 4$$

14. If $2y^2\frac{dx}{dy} + 2xy + x^2 = 0$, $y > 0$, where $x = x(y)$ and $x(e) = e$, then the value of $x(e^2)$ is

(1) $\frac{4e^2}{5e^2+1}$ (2) $\frac{4e^2}{5e^2-1}$

(3) $\frac{5e^2}{4e^2+1}$ (4) $\frac{5e^2}{4e^2-1}$

Answer (2)

Sol. $2y^2\frac{dx}{dy} + (2xy + x^2) = 0$

$$\frac{2y^2}{x^2}\frac{dx}{dy} + \frac{2y}{x} = -1$$

Put $\frac{1}{x} = t \Rightarrow \frac{dt}{dy} = -\frac{1}{x^2}\frac{dx}{dy}$

$$\Rightarrow -\frac{dt}{dy}(2y^2) + 2yt = -1$$

$$\Rightarrow \frac{dt}{dy} + \left(-\frac{1}{y}\right)t = \frac{1}{2y^2}$$

$$\text{I.F.} = e^{\int -\frac{1}{y}dy} = \frac{1}{y}$$

$$\Rightarrow t\cdot\frac{1}{y} = \int\frac{1}{2y^3}dy + C$$

$$\Rightarrow \frac{t}{y} = -\frac{1}{4y^2} + C$$

$$\Rightarrow \frac{1}{xy} = -\frac{1}{4y^2} + C$$

$\therefore x(e) = e$

$$\Rightarrow \frac{1}{e^2} = -\frac{1}{4e^2} + C \Rightarrow C = \frac{5}{4e^2}$$

$$\Rightarrow \frac{1}{xy} = -\frac{1}{4y^2} + \frac{5}{4e^2}$$

$$\Rightarrow \frac{1}{x(e^2)e^2} = -\frac{1}{4e^4} + \frac{5}{4e^2} \Rightarrow \frac{1}{x(e^2)} = -\frac{1}{4e^2} + \frac{5}{4}$$

$$\Rightarrow \frac{1}{x(e^2)} = \frac{-1+5e^2}{4e^2} \Rightarrow x(e^2) = \frac{4e^2}{5e^2-1}$$

15.

16.

17.

18.

19.

20.

SECTION - B

Numerical Value Type Questions: This section contains 5 Numerical based questions. The answer to each question should be rounded-off to the nearest integer.

21. The value of the sum $\sum_{n=1}^{8}\frac{1^3+2^3+\ldots.\ +\text{upto } n \text{ terms}}{1+3+5+\ldots\text{upto } n \text{ terms}}$ is

Answer (71.00)

Sol. $$\sum_{n=1}^{8}\frac{\left(\frac{n(n+1)}{2}\right)^2}{n^2} = \sum_{n=1}^{8}\frac{n^2(n+1)^2}{4n^2}$$

$$= \sum_{n=1}^{8}\frac{(n+1)^2}{4} = \sum_{n=0}^{8}\frac{(n+1)^2}{4} - \frac{1}{4}$$

$$= \frac{1^2+2^2+\ldots+8^2+9^2}{4} - \frac{1}{4}$$

$$\frac{9\times10\times19}{6\times4} - \frac{1}{4} = 71$$

22. $f(x)$ is 5 degree polynomial has extremes at $x = \pm 1$ and $\lim_{x\to 0}\frac{f(x)}{x^3} = -5$, then $f(2) - f(-2)$ is

Answer (112)

Sol. $f'(x) = (x-1)(x+1)(ax^2 + bx + c)$

$$f'(x) = (x^2-1)(ax^2+bx+c)$$

$$f'(x) = ax^4 + bx^3 + cx^2 - ax^2 - bx - c$$

$$f'(x) = ax^4 + bx^3 + (c-a)x^2 - bx - c$$

$$f(x) = \frac{ax^5}{5} + \frac{bx^4}{4} + \frac{(c-a)x^3}{3} - \frac{bx^2}{2} - cx + d$$

$\lim_{x\to 0}\frac{f(x)}{x^3} = -5$ Then $b = c = d = 0$

and $\frac{c-a}{3} = -5$

$a = 15$

$$f(x) = 3x^5 - 5x^3$$

$$f(x) - f(-x) = f(x) + f(x)$$

$= 2f(x)$

$= 2[3(2)^5 - 5(2)^3]$

$= 112$

23. If ${}^{30}C_{30-r} + 3\left({}^{30}C_{31-r}\right) + 3\left({}^{30}C_{32-r}\right) + {}^{30}C_{33-r}$ $= {}^{m}C_r \forall r \in \{0, 1, \ldots 30\}$ then m is equal to

Answer (33.00)

Sol. $\left({}^{30}C_{30-r} + {}^{30}C_{31-r}\right) + 2\left({}^{30}C_{31-r} + {}^{30}C_{32-r}\right) + {}^{30}C_{32-r} + {}^{30}C_{33-r}$

using ${}^{n}C_{r-1} + {}^{n}C_r = {}^{n+1}C_r$

$${}^{31}C_{31-r} + \left({}^{31}C_{32-r}\right) + {}^{31}C_{33-r}$$

$$= {}^{31}C_{31-r} + {}^{31}C_{32-r} + {}^{31}C_{32-r} + {}^{31}C_{33-r}$$

$$= {}^{32}C_{32-r} + {}^{32}C_{33-r} = {}^{33}C_{33-r}$$

$$= {}^{32}C_r = {}^{m}C_r$$

$\Rightarrow m = 33$

24. If P_n denotes the number of triangles formed by the vertices of n-sides polygon and $P_{n+1} - P_n = 66$, then n is

Answer (12)

Sol. ${}^{n+1}C_3 - {}^{n}C_3 = 66$

$(n+1)\cdot n(n-1) - n(n-1)(n-2) = 66 \times 6$

$\Rightarrow \boxed{n = 12}$

25. Let $f(x) = \left[x^2 - x - \frac{1}{2}\right]$, then the number of points of discontinuity in [2, 4] is/are (Where $[\cdot]$ denotes greatest integer function)

Answer (10)

Sol. Let $g(x) = x^2 - x - \frac{1}{2}$

$g(x)$ is $\uparrow$ing in [2, 4]

$\Rightarrow \quad g(x) \in [f(2),\ f(4)]$

$$\Rightarrow g(x) \in \left[\frac{3}{2}, \frac{23}{2}\right]$$

$\therefore f(x)$ is discontinuity wherever $f(x)$ will become integer.

$\Rightarrow g(x)$ is discontinuous if $g(x) = 2, 3, 4, \ldots, 11$

So total 10 points of discontinuity

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